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# Inhomogeneous diffusion process in elongated rooms: An interpretation based on the dynamics of sound particles

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**Abstract:** Previous studies showed that the reverberant field in elongated rooms is governed by non-homogeneous diffusion. The objective of this study is to physically interpret this phenomenon by considering the dynamics of the sound particles. Starting from the original diffusion theory, a quantity that can be interpreted as a “local” mean free path has been proposed and computed from the paths of the propagating particles. Based on the proportionality relationship between the mean free path and the diffusion coefficient, the spatial distribution of the latter could be estimated and successfully compared with a direct estimation using the Fick’s law. © 2021 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).

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## 1. Introduction

Assuming that the sound field can be seen as a set of elementary energy sound particles propagating at the speed of sound, this work concerns the study of the dynamics of these particles when the room under study is of elongated shape. For several years now, the original theory of diffusion, modeling the propagation of diffusing particles through scattering targets (presumed infinite medium), has been adapted to the case of room acoustics (acoustic diffusion model for a closed environment).<sup>1</sup> The acoustic energy density, corresponding to the number of sound particles in an elementary volume, is then assumed to be governed by a diffusion equation, a key factor being a diffusion coefficient accounting for the impact of the room on the diffusion process. In the original theory,<sup>2</sup> this diffusion coefficient can be spatially dependent due to its proportionality to the mean free path, which can vary spatially and can be expressed as the distance travelled by the particles without collisions with targets weighted by a non-collision probability. Currently, this mean free path is chosen to be equal to the mean free path of the room, which involves a homogeneous diffusion process (constant diffusion coefficient). However, this last expression is not valid in the case of elongated rooms, as has been shown from the Fick’s law based on acoustic energy density and intensity.<sup>3</sup> The diffusion coefficient increases with the distance to the sound source and also according to the wall absorption and room aspect ratio, involving an inhomogeneous diffusion process. Acoustics predictions of reverberation in rooms with large aspect ratios turn out to be erroneous if the inhomogeneity of diffusion is not taken into account.

The goal of this study is to understand how to relate the propagation of elementary sound particles to the diffusion coefficient associated with inhomogeneous diffusion. The work will be based on a particle-tracing code using a Monte-Carlo approach. A “local” mean free path dedicated to elongated rooms will be defined, inspired by that of the classical theory of diffusion<sup>2</sup> and taking advantage of the one-dimensional geometry of long rooms. This approach is justified by the fact that the elongated room is likely to act as a guide for the paths of the sound particles; the distances travelled by the particles should then increase as the source–receiver distance increases. Assuming that the proportionality relationship between the diffusion coefficient and the mean free path is respected locally, an estimation of the space-varying diffusion coefficient based on the mean free path will be obtained and compared with the value based on the Fick’s law of diffusion to validate the model.

Section 2 briefly reviews diffusion theory and then focuses on the diffusion coefficient and its proportionality with the mean free path. Section 3 summarizes previous work based on the Fick’s law and pointing out the necessity of

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introducing inhomogeneous diffusion for the case of elongated rooms (spatially varying diffusion coefficient). Section 4 presents the particle-tracing approach used to estimate and record the paths travelled by sound particles during their propagation in the room. The expression of the local mean free path based on these paths and then that of the diffusion coefficient are given in Sec. 5. A validation study is carried out in Sec. 6, followed by the conclusion (Sec. 7).

## 2. Diffusion coefficient and mean free path

The acoustic diffusion model is based on the hypothesis that the reverberant sound field can be seen as a set of sound particles of elementary energy propagating along straight lines between two successive collisions with the walls.<sup>1</sup> This propagation is assumed to be similar to that of particles propagating through an equivalent volumetric scattering medium made of targets by multiple collisions (such as light propagation through fog). In this modeling framework, the acoustic energy density  $w(\mathbf{r})$ , corresponding to the number of sound particles in an elementary volume centered at  $\mathbf{r}$  (the space variable), is governed by a diffusion equation, associated with the ‘‘Fick’s law,’’ expressing the proportionality between the intensity  $\mathbf{I}(\mathbf{r})$  and the energy density gradient  $\nabla w(\mathbf{r})$ .<sup>2</sup> The accuracy of room-acoustic predictions depends on the diffusion coefficient, possibly spatially varying and denoted  $D(\mathbf{r})$ . It has been shown that the diffusion coefficient is proportional to the speed of sound  $c$  and the mean free path  $\lambda(\mathbf{r})$  as

$$D(\mathbf{r}) = \frac{c \lambda(\mathbf{r})}{3}. \tag{1}$$

The mean free path  $\lambda(\mathbf{r})$ , which corresponds to the mean distance travelled by the diffusing particles between two collisions with scattering targets, is obtained in the original diffusion theory (the scattering medium is infinite) from the following expression:<sup>2</sup>

$$\lambda(\mathbf{r}) = \int_0^{+\infty} l dP(\mathbf{r}, l), \tag{2}$$

where  $dP(\mathbf{r}, l) = S_V(\mathbf{r})P(\mathbf{r}, l)dl$  is the probability of not colliding when traveling the distance between  $l$  and  $l + dl$  in an area centered on position  $\mathbf{r}$ , and  $S_V(\mathbf{r})$  is the local scattering section per unit volume of the equivalent scattering medium. The probability  $P(\mathbf{r}, l)$  is exponential, i.e.,  $P(\mathbf{r}, l) = \exp(-S_V(\mathbf{r})l)$ . The mean free path could then be expected to change spatially if the local amount of scattering targets changes. The calculation of this integral shows that the mean free path is equal to  $1/S_V(\mathbf{r})$ .

In the analogy with room acoustics derived by Picaut *et al.*,<sup>1</sup> it was proposed that the scattering medium equivalent to the room is made of spheres of constant radius, homogeneously located in the volume of the room. This hypothesis implies that the probability of a sound particle undergoing a collision is identical whatever its position and direction of propagation. It followed that the scattering section per unit volume  $S_V$  is equal to  $S/4V$  ( $S$  and  $V$  being, respectively, the total surface area and volume of the room). Consequently, the corresponding mean free path  $1/S_V$  is equal to the classical mean free path in room acoustics, denoted  $l_c = 4V/S$ . The diffusion coefficient  $D_c$  is then constant in the context of this theory, according to

$$D_c = \frac{l_c c}{3}, \tag{3}$$

and the diffusion process is therefore homogeneous.

## 3. The case of elongated rooms

Elongated rooms are known to produce significant spatial variations of the reverberant field and have been studied using transport or diffusion theories.<sup>3,4</sup> Visentin *et al.*<sup>3</sup> have focused their work on this type of configuration to perform numerical estimates of the diffusion coefficient using the Fick’s law. With a particle-tracing code,<sup>5</sup> the spatial distribution of the acoustic energy density gradient and of the acoustic intensity vector was predicted. In the case of an elongated room, it is supposed that the quantities vary essentially along the longest dimension of the room (the corresponding axis is  $OX$ ), and, from the Fick’s law, one can estimate the ‘‘local’’ diffusion coefficient at position  $X$ ,

$$D_{\text{Fick}}(X) = \frac{\|\mathbf{I}(X)\|}{\|\nabla w(X)\|}. \tag{4}$$

In this work, the estimation of  $D_{\text{Fick}}(X)$  using the Fick’s law was considered as a numerical ‘‘measurement’’ of the diffusion coefficient variation. For the case of weakly absorbent corridors, Visentin *et al.*<sup>3</sup> showed that the diffusion coefficient  $D_{\text{Fick}}$  varies weakly with space and that the expression proposed in Eq. (3) remains practically valid. Conversely, when the wall absorption increases, the diffusion process is non-homogeneous. The diffusion coefficient  $D_{\text{Fick}}$  increases as a function of the position  $X$ , the absorption  $\alpha$ , and the cross section of the corridor  $S_c$ . In this case, the predictions obtained using the homogeneous diffusion model (i.e., constant diffusion coefficient  $D_c$ ) indicate that the spatial decrease

in the reverberant sound level can be highly overestimated for rooms with large aspect ratios. Conversely, by taking into account non-homogeneous diffusion in the diffusion model predictions, this spatial decay is correctly predicted.<sup>3</sup>

#### 4. Investigation of the dynamics of sound particles

##### 4.1 The sound-particle technique

The particle-tracing software used in this work is based on a Monte-Carlo method.<sup>3,5</sup> A sound source emits a large number of sound particles colliding with the walls. The influence of the wall absorption  $\alpha$  is taken through the draw of a random number  $u$  between 0 and 1 at each collision. If  $u \geq \alpha$ , the sound particles are absorbed by the wall. In this way, absorption reduces the number of sound particles propagating and therefore has an influence on the overall dynamics of the particles. In all simulations, wall reflections are defined as entirely diffused, using Lambert’s law.

During the simulation, the particle propagation history is stored, in particular the positions of their collisions with the walls [Fig. 1(a)]. Let us define a path  $L_i$  as a vector relating two collision points. A set of  $N$  paths  $L_i$  ( $1 \leq i \leq N$ ) is obtained for a given simulation, describing all the paths covered by the simulated particles. The path length is the norm of  $L_i$  and is denoted  $L_i$ .

In the following, the hypothesis made by Visentin *et al.*,<sup>3</sup> according to which the phenomena in an elongated room are essentially one-dimensional, is retained, implying that the diffusion coefficient varies only along the main axis  $OX$  of the room. Thus, the considered cross sections are perpendicular to  $OX$ . The set of paths crossing a disk of radius 1 m contained in a cross section located at  $X$  is selected [denoted  $L_i(X)$  and highlighted in red in Fig. 1(a)]; the number of these paths is denoted  $N(X)$ .

##### 4.2 Particle path investigation

The angle  $\theta_i$  formed by  $L_i$  and  $\mathbf{n}$  is now studied ( $\cos(\theta_i) = \mathbf{n} \cdot L_i/L_i$ ),  $\mathbf{n}$  being a unit vector parallel to the axis  $OX$  and pointing toward the direction of propagation of the particle, so that  $\theta_i$  is in the range  $[0;90]^\circ$  [Fig. 1(b)]. The angle  $\theta_i(X)$  is then an indicator of the deviation of the particle path  $L_i(X)$  from the main axis  $OX$  of the room, at a position  $X$ . A value close to  $0^\circ$  indicates that the particle follows a path nearly parallel to the room main axis  $OX$ . The averages of the angles and lengths of the paths are calculated and noted, respectively, as  $\bar{L}(X) = \sum_{i=1}^N L_i(X)/N(X)$  and  $\bar{\theta}(X) = \sum_{i=1}^N \theta_i(X)/N(X)$ .

Results are presented in Fig. 2 for an elongated room of dimensions  $[40, 4, 4] \text{ m}^3$  and different absorption coefficients  $\alpha$ . The end walls at  $X=0 \text{ m}$  and  $X=40 \text{ m}$  are made fully absorbent ( $\alpha=1$ ) to attenuate end effects, while the absorption of the lateral walls varies. The source is located at the center of the corridor cross section at position  $X=2 \text{ m}$ . The number of sound particles  $N_0$  varies from  $200 \times 10^3$  to  $600 \times 10^3$ , depending on the configuration to ensure statistical convergence. The disks are positioned along the  $OX$  axis equidistantly, with a spacing equal to 0.25 m.

Close to the source, the average of the paths  $L_i$  is close to the theoretical mean free path  $l_c$ , showing that the reverberant sound field is still isotropic and in agreement with diffuse sound field theory [Fig. 2(a)]. Moving away from the source, the average length of paths is increasing. These results are corroborated by Fig. 2(b). When the  $X$  position gets further from the source,  $\bar{\theta}$  gets smaller, meaning that the direction of the paths  $L_i$  tends to be parallel to the  $OX$  axis of the elongated room. The room acts like a “guide” modifying the behavior of the reverberant sound field, which is evidenced by the trend of the particle of using paths more and more parallel to the room axis  $OX$  and therefore of increasing length. As expected, Fig. 2 shows that this trend is accentuated when the room absorption increases.

#### 5. A “local” mean free path

This behavior could be the explanation for the increase in the diffusion coefficient in long rooms (Sec. 3). Indeed, the previous results indicate that at positions far from the source, the sound particles statistically tend to propagate parallel to the main axis  $OX$ , which may facilitate the diffusion of energy away from the source. Diffusion theory states the

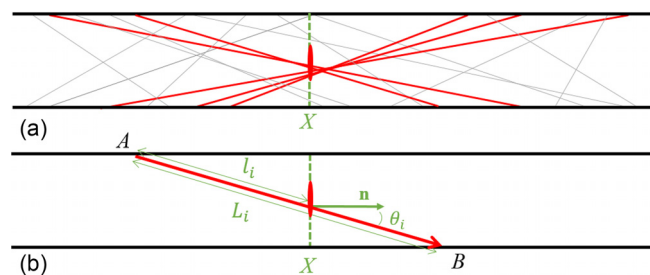


Fig. 1. (a) Two-dimensional representation of the set of paths traveled by sound particles during their propagation within the room; in red are highlighted the paths  $L_i(X)$  crossing a disk in the center of the cross section located at  $X$ . (b) Path  $L_i$  of length  $L_i$  between two successive collision points  $A$  and  $B$ .  $L_i(X)$  is defined as the distance between the starting collision point  $A$  and the cross section  $X$  and forming an angle  $\theta_i$  with the normal  $\mathbf{n}$  of the cross section in the direction of propagation of the sound particle.

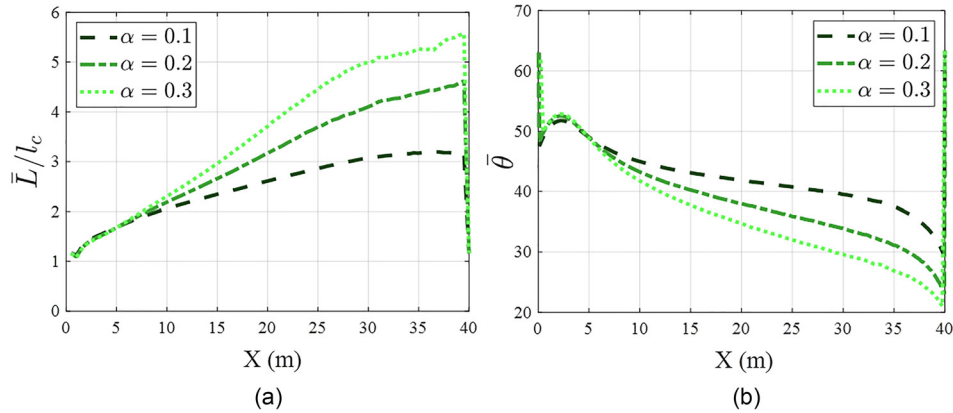


Fig. 2. Case of the elongated room [40:4:4] associated with different absorption values at the lateral walls: Evolution of the normalized mean length  $\bar{L}(X)/l_c$  (a) and of the mean angle  $\bar{\theta}$  (b) as a function of the position  $X$ .

proportionality of the diffusion coefficient with the mean free path [Eq. (3)]. The hypothesis that the spatial increase in the diffusion coefficient  $D_{\text{Fick}}(X)$  observed by Visentin *et al.*<sup>3</sup> may be the result of an increase in a local statistical quantity having the dimension of a length, i.e., a “local” mean free path, is then made. To explore this hypothesis, this local mean free path is determined from a Monte-Carlo method,<sup>6,7</sup> using the sound-particle technique described in Sec. 4.1. Similar investigations of the spatial distribution of the mean free path can be found in the literature for the case of strongly rarefied gas inside a cylinder in the Knudsen regime.<sup>8</sup>

Recalling the general definition of the mean free path of the diffusion theory in Eq. (2), for each recorded path  $L_i$ , the length  $l_i(X)$  of the path when it passes through the cross section located at  $X$  is defined [Fig. 1(b)]. Thus, similarly to the distance  $l$  corresponding to the distance travelled without collision in a supposedly infinite volumetric medium centered at  $\mathbf{r}$  [Eq. (2)], the distance  $l_i(X)$  corresponds to the distance travelled without collision before crossing the cross section located at  $X$  in the finite volumetric medium that is the room.

From the obtained catalog of paths, a quantity analog to  $l dP(X, l)$  of Eq. (2) is required ( $\mathbf{r}$  being replaced by  $X$  for a one-dimensional problem), so that the path  $l$  covered by the particle is approached by the length  $l_i(X)$ , weighted by a model of the probability  $p_i(X)$  of the particle not to be absorbed through its propagation (from the starting point to the end point of the path). The exponential probability mentioned in Sec. 2 is not a good candidate, because it is valid for an infinite scattering medium and the room is a finite medium. A weighting function  $p_i(X)$  that is equal to 1 at the start of the ray [the particle has not collided,  $l_i(X) = 0$ ] and equal to 0 at the collision point [the particle has collided,  $l_i(X) = L_i$ ] is required. Assuming a linear variation in between, the above function  $p_i(X)$  is proposed,

$$p_i(X) = 1 - \frac{l_i(X)}{L_i}. \tag{5}$$

In this way, as shown in Fig. 1(b), when the particle going from point  $A$  to point  $B$  passes through the disk located at  $X$ , its probability of not having collided through the equivalent volumetric scattering medium is modeled by  $p_i(X)$ . Finally, to perform an average of the  $N$  samples of  $l_i(X)p_i(X)$ , a weighted sum  $\Lambda(X)$  is calculated to compute a quantity that could be interpreted as a “local” mean free path,

$$\Lambda(X) = \left( \sum_{i=1}^N l_i(X)p_i(X) \right) / \left( \sum_{i=1}^N p_i(X) \right). \tag{6}$$

An estimate of the local diffusion coefficient  $D_p(X)$ , based on the dynamics of sound particles, is then obtained using Eq. (1), i.e.,  $D_p(X) = c\Lambda(X)/3$ .

## 6. Validation

To validate the proposed expression of Eq. (6), the diffusion coefficient  $D_p(X)$  based on the sound particle dynamics is compared to the one estimated for the Fick’s law [the quantity  $D_{\text{Fick}}(X)$  in Eq. (4)]. For this purpose, the energy intensity and density were estimated using the energy prediction model based on the same particle-tracing software,<sup>5</sup> at the same points as in Sec. 4 (every 0.25 m). The receiver volumes used for calculating the energy density and intensity around these points are spherical, with a radius of 1 m to match the radius of the disks used for path selection. The resulting partial overlap of these receiving volumes allows us to obtain smoothed intensity and density curves, facilitating in particular the estimation of the gradient in Eq. (4) by a central finite difference scheme.<sup>3</sup> Results are provided for different absorption coefficients and two rooms of dimension [40:4:4] and [40:3:3] m<sup>3</sup>, respectively, in Figs. 3(a) and 3(b). The source is at

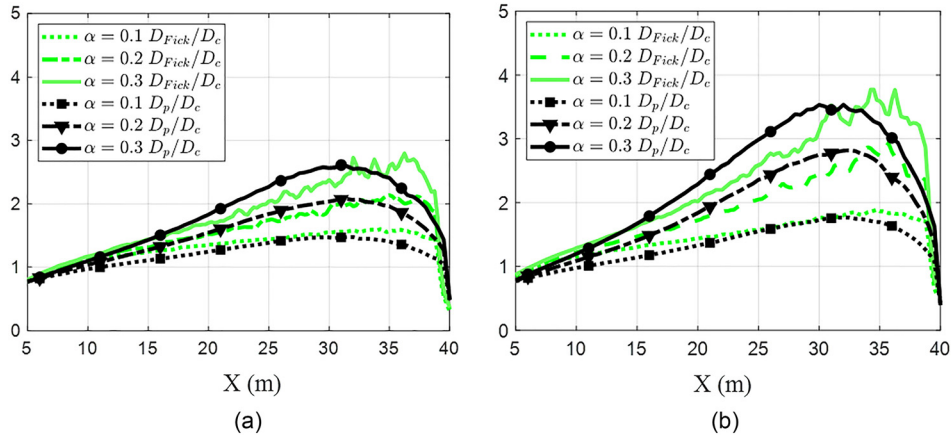


Fig. 3. Comparison between  $D_{\text{Fick}}(X)/D_c$  and  $D_p(X)/D_c$  for an elongated room of dimensions (a) [40: 4: 4] m<sup>3</sup> and (b) [40: 3: 3] m<sup>3</sup> and different absorption coefficients. The source is at the center of the cross section at position  $X=2$  m. The end walls at  $X=0$  m and  $X=40$  m are made fully absorbent ( $\alpha=1$ ).

the center of the cross section at position  $X=2$  m. For an easier interpretation, the obtained diffusion coefficients are normalized by the diffusion coefficient  $D_c$  of the original theory [Eq. (3)].

The agreement between  $D_p(X)$  and  $D_{\text{Fick}}(X)$  is rather satisfactory for all geometries and absorption coefficients. The diffusion coefficient estimated from the particle dynamics retrieves the expected trends with an increase in the diffusion coefficient as a function of the position and of the absorption, and the rate of increase is higher when absorption increases in the same proportions. Similar results (not shown in this paper) were also observed when the source was positioned midway between the end walls. It is also remarkable that the effect of the absorbing end wall, i.e., the drop of the diffusion coefficient, is well modeled by the particle dynamics approach. The relative error between the two models has a maximum value of 17% for the room of dimensions [40: 4: 4] but increases when the aspect ratio is very high (dimensions [40: 3: 3]) with a maximum value of 21%. A trend of the influence of absorption on the relative error does not appear clearly.

## 7. Conclusion

The objective of this study was to interpret non-homogeneous diffusion in elongated rooms by considering the dynamics of the sound particles propagating in the room. Starting from the original diffusion theory, a quantity that can be interpreted as a local mean free path has been proposed and computed from the paths of the propagating particles using a Monte-Carlo approach. Based on the proportionality relationship between the mean free path and the diffusion coefficient, the spatial distribution of the latter could be estimated and successfully compared with a direct estimation using the Fick's law for two elongated rooms and a range of absorption coefficients. To conclude, this work provides a better understanding of the dynamics of sound particles in the case of inhomogeneous diffusion in elongated rooms. The particle path statistics are used to propose a semi-analytical expression relating this dynamics to the diffusion coefficient of the acoustic diffusion model. Ultimately, this work should contribute to the establishment of a purely analytical expression of the diffusion coefficient, allowing the use of the acoustic diffusion model for configurations generating significant inhomogeneous diffusion effects (elongated rooms, flat rooms).

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